

An Sample Analysis of Heavy Metal Pollution to Urban Surface Soil Based on Transfer Function Theory

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Abstract. The degree of heavy-metal pollution of different district in city area is discussed by improved Nemerow model and further topographic maps are drawn. From the improved Nemerow model, the Nemerow integrated pollution index is determined. The main reason of pollution is found according to weight, then the corresponding heavy-metal pollution things portion in the overall district is computed by using the model. The element that corresponds the maximum weight is found. And it proves that the heavy-metal pollution element Hg is the main pollution reason. The pollution source is determined by heavy metal spreading features using transfer function theory. Thinking of the main pollution elements as the researched object, the spreading features are analyzed using transfer function characteristics. And the spreading process is described using flow chart.

Introduction

City is the important place of human production and life, the urban soil constitutes is one of the main parts in urban environment. Frequent human activities and intensive industries, transportation Make the urban soil suffer strong anthropogenic interference Cause serious heavy metal pollution. Today, it is speed up In the urbanization process, The urban soil heavy metal not only the complicated sources, and quantity, Heavy metal to the human damage way also become more complicated, and the city is human activity concentration areas, so the urban soil heavy metal pollution on human health threat more, it is should get people's attention. At present, to the city of heavy metal pollution propagation characteristics of the system have not been formed, mature, theoretical system. The spread of heavy metal pollution evaluation method of single factor index quality evaluation, hierarchical analysis and neural network and the matter-element analysis and so on. In this article, the Nemerow index model, fuzzy mathematical weights model, the transfer function model, feedback model of surface soil heavy metal pollution of the city are analyzed eight major of the heavy metal element in the urban spatial distribution, and analyzes the city, the different areas of heavy metal pollution levels. That is the main reason for the heavy metal pollution. And heavy metal pollutants analysis of the propagation characteristics of established, then the model, to determine the position of the pollution sources, and to better serve the study of urban geological environmental services [1-4].

Nemerow Improved Model

One of the Eight Major Heavy Metal Elements in the City of The Spatial Distribution and of the Established. Use Matlab to do the sampling points, the axis and elevation of the scatterplot chart, and the use of different points to mean different sampling area. (note: * represents a first class area, o represents a second class area, + represents a third class area, represents a fourth class area, represents a fifth class area).

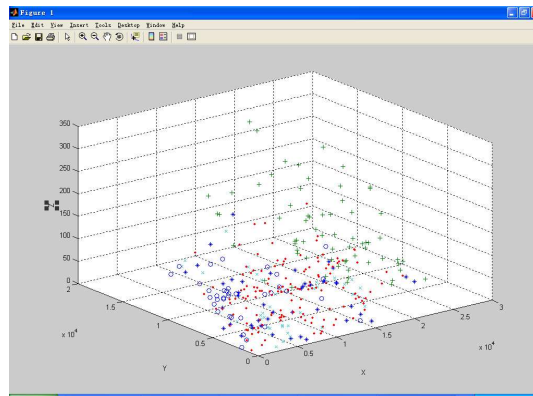


Fig. 1 Heavy metal elements in the city of the spatial distribution

Analysis of Heavy Metal Pollution Levels in Different Areas

Nemerow index method is the comprehensive pollution index calculation at home and abroad and one of the most common methods of is a kind of extreme value or outstanding of both the maximum sum right type factor environment quality index. The calculation formula is:

$$P_{ij} = C_{ij} / S_{ij} \quad (i = 1,2,3,4,5 \quad j = 1,2,3,4,5,6,7,8) \quad P = \left[\frac{(P_{ij})^2_{\max} + (P_{ij})^2_{\text{ave}}}{2} \right]^{\frac{1}{2}}$$

Table 1 District comprehensive pollution index (P)

area	$(P_{ij})^2_{\max}$	$(P_{ij})^2_{\text{ave}}$	P
1	14.00754347	6.097826923	3.170596978
2	336.8329003	29.45212921	13.53301573
3	1.721119919	1.419602111	1.253140461
4	162.9799011	14.7346942	9.426414889
5	10.79436274	4.159094651	2.734360747

Table 2 Nemerow comprehensive pollution index rating criteria

	P	Pollution levels	Pollution degree
1	$P < 0.7$	security	clean
2	$0.7 < P < 1.0$	cordon	A little clean
3	$1.0 < P < 2.0$	Light pollution	Exceeds bid
4	$2.0 < P < 3.0$	Middle pollution	Soil and crop by moderate pollution
5	$P > 3.0$	Heavy pollution	By severe pollution of soil, the crops

So, according to table 1, it is known that the three kinds of area is light pollution, five class area in the area is pollution, one class area, two class area and four class area is heavy pollution and pollution degree is the second class area > four class area > one class area > five class area > the third class area.

Weight Model [5-8]

Assessment of heavy metal contamination of the weight of each element formula:

$$W_{ij} = \frac{\frac{C_j}{S_j}}{\sum_{j=1}^8 \frac{C_j}{S_j}}$$

Type : Heavy Metal Pollutants in the Area Accounts for the Weight

Table 3 Data analysis results

function	As ($\mu\text{g/g}$)	Cd (ng/g)	Cr ($\mu\text{g/g}$)	Cu ($\mu\text{g/g}$)	Hg (ng/g)	Ni ($\mu\text{g/g}$)	Pb ($\mu\text{g/g}$)	Zn ($\mu\text{g/g}$)	$W_{ij \max}$	A_i
	Weight1	Weight 2	Weight 3	Weight 4	Weight 5	Weight 6	Weight 7	Weight 8	Weight max	The metal
1 Living areas	0.088169	0.112906	0.112700	0.189453	0.134563	0.075486	0.112844	0.173875	0.189453	Cu
2 Industrial areas	0.046394	0.069650	0.039683	0.222540	0.422725	0.037099	0.069129	0.092775	0.422725	Hg
3 Mountain areas	0.117854	0.122924	0.131850	0.137636	0.122765	0.131812	0.123715	0.111441	0.137636	Cu
4 Traffic areas	0.051632	0.090181	0.060983	0.153482	0.415725	0.046641	0.066739	0.114613	0.415725	Hg
5 Park green areas	0.106644	0.132271	0.086276	0.140192	0.201376	0.076191	0.120032	0.137014	0.201376	Hg
6 The whole city	0.063912	0.094285	0.069964	0.168939	0.347092	0.056884	0.080727	0.118193	0.347092	Hg

The weight of the largest district corresponding heavy metal pollutants A_i ($i = 1,2,3,4,5$)

Living areas: A_1 --- Cu ; Industrial areas: A_2 --- Hg , Mountain areas: A_3 --- Cu ;

Traffic areas: A_4 --- Hg ; Park green areas: A_5 --- Hg ;

Through to the analysis of the form 1-3 results, in five area the three of the weight of the maximum referring to the elements are Hg , the whole city was one of the largest weights is Hg , so heavy metal pollution is Hg ,it is the main reason of the elements, that the heavy metal pollution is mainly in the industrial zone and the traffic area.

The Transfer Function Model

The transfer function [3] is through the system output and input of the relationship between the inherent characteristics to describe system, that system to the external characteristics of the system to reveal the interior characteristics; this is the basic idea of the transfer function. The transfer function concept and basic ideas in signal and system which has special meaning, when a system structure is not clear, or can't figure out how it's internal structure, with the system input to see from the output of the system, can also study the function of the system and natural characteristics.

Specific model and analysis are as follows:

The Establishment of The Function [9-13]. First of all, set the maximum concentration point for space coordinates (x_1, y_1, z_1) ,

Seek the other point and the maximum concentration point distance

$R = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}$, And to find out all points of the corresponding Hg concentrations, Using *Matlab* simulation to produce five area function of image, and get every area function $f_i(x)$

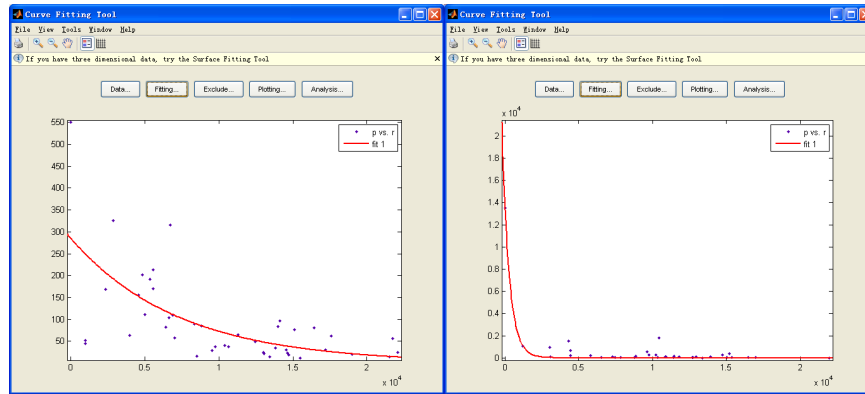


Fig. 2 $f_1(x)$ picture

Fig. 3 $f_2(x)$ picture

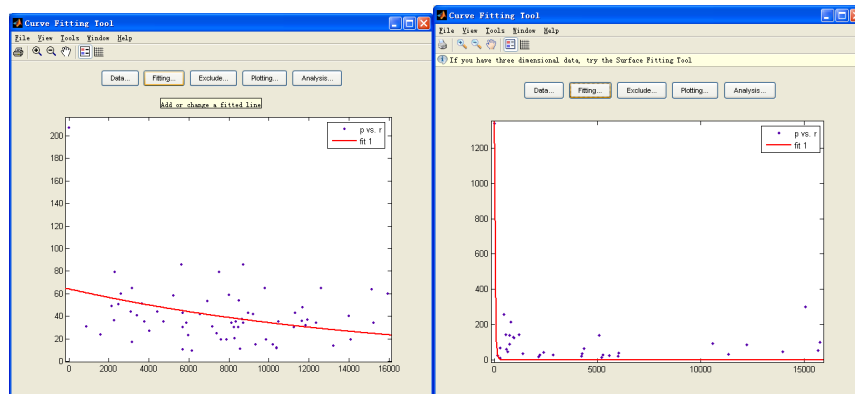


Fig. 4 $f_3(x)$ picture

Fig. 5 $X_1 \rightarrow X_0$ picture

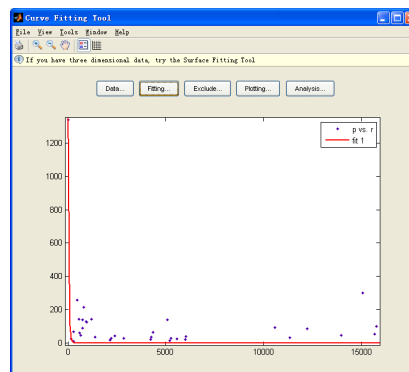


Fig. 6 $f_5(x)$ picture Get:

$$f_1(x) = 286 \times e^{-0.00001371x} \quad f_2(x) = 13500 \times e^{-0.002059x}$$

$$f_3(x) = 50.32 \times e^{-0.02915x} + 18.74 \times e^{-0.002059x}$$

$$f_4(x) = 16000 \times e^{-0.009618x} \quad f_5(x) = 1339 \times e^{-0.02312x}$$

4.2 $f(x)$ of the Laplace transform $F_i(s)$

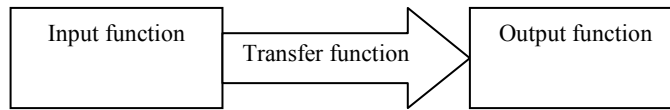
According to the signal and system of Laplace basic formula $F(s) = \int_0^\infty f(t)e^{-st} dt$ for the five areas, $F_i(s)$.

$$F(s) = \frac{286}{s + 0.0001371} \quad F_2(s) = \frac{13500}{s + 0.002059}$$

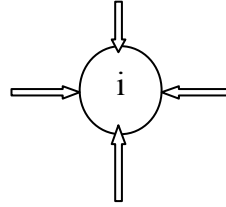
$$F_3(s) = \frac{50.32}{s + 0.02915} + \frac{18.74}{s + 0.00001634}$$

$$F_4(s) = \frac{1600}{s + 0.009618} \quad F_5(s) = \frac{1339}{s + 0.02312}$$

Spread Flowchart



Drawing Propagation Characteristics Map



By the other districts spread to the i:

Seek to Pass a Total Function. Pass the total function is all the sum of the single district of , that is,

$$h_j(x) = \sum_{i=1}^5 g_{ij}(x) \quad (j=1,2,3,4,5)$$

$$h_1(x) = 0.2526528 \times \delta(x) + 0.0032511967318 \times e^{-0.0001317x}$$

$$h_2(x) = 69 \times \delta(x) + 25.2 \times e^{-0.002059x}$$

$$h_3(x) = 0.258 \times \delta(x) - 0.000323161 \times e^{-0.02915x} + 0.000345318 \times e^{-0.00001634x}$$

$$h_4(x) = 57.9259018 \times \delta(x) - 1.97267 \times e^{-0.009618x}$$

$$h_5(x) = 4.8 \times \delta(x) + 0.048153 \times e^{-0.02312x}$$

4.6 Using draw the five functions of graphics

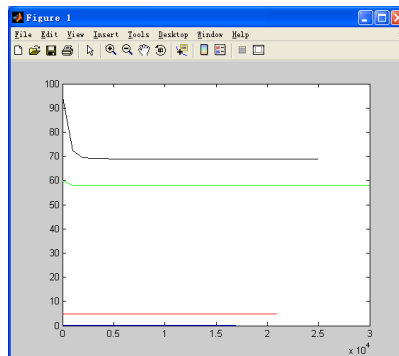


Fig. 7 Function graph

Followed by expression of the function image from top to bottom, Industrial Zone, Traffic Zone , Living area, Park Greenbelt areas, Mountain, However, due to the low mountain curves, So was not obvious in Fig.

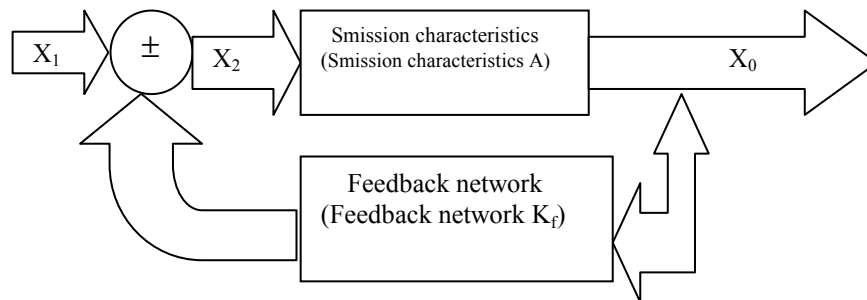
Analysis and Conclusions [14-16]. According to the characteristics of the transfer function, transfer function model, this model expresses the propagation characteristics of heavy metal pollutants, and thus determine the location of pollution sources, namely sources of pollution than traffic pollution in industrial areas and traffic areas, including industrial areas District is slightly larger. Get the size of the transmission intensity of pollutants according to the function of $h_j(x)$ graphics, that is: Industrial Zone>Traffic Zone>Living area>Park Greenbelt>Mountain, This conclusion is consistent with the conclusions of problem, which can show that this transfer function model is reasonable.

Feedback Network Model

With population growth, industrial factories in the increase, affecting the soil concentration of heavy metal pollution of the geological environment with the function of the distance change, so the need to

collect the past 10 years the city the same sampling point eight kinds of heavy metal pollutant concentration values, population growth relations, and the concentration of industrial activity and concentration and so on.

Spread function model based on the establishment of its dissemination of feedback model, than is:



Get parameters A , K_f and Input signal (X_1, X_2) , The relationship

Between Output signal (X_0) feedback signal X_f

$$X_0 = A \times X_2 \quad (1) \quad X_f = K_f \times X_0 \quad (2) \quad X_1 = X_2 + X_f \quad (3)$$

Definition of closed-loop gain is the actual transmission rate of pollution define the feedback depth, and equation (1) (2) (3) into

$$A_f = \frac{X_0}{X_1} = \frac{A}{1 + \frac{X_f}{X_2}} \quad F = 1 + \frac{X_f}{X_2} = 1 + \frac{X_f}{X_0} \times \frac{X_0}{X_2} = 1 + A \times K_f \quad A_f = \frac{A}{F}$$

From the previous analysis:

- ① When the feedback factor $K_f < 0$, the system is negative feedback, and heavy metal pollutants propagation characteristics of slow spread from the $X_1 \rightarrow X_0$.
- ② When the feedback factor $K_f > 0$, the system is a positive feedback, heavy metal pollutants propagation characteristics for the rapid spread of the $X_1 \rightarrow X_0$.

Summary: Evaluation and Improvement of the Model

Advantage. Use of electronic information engineering expertise to comprehensive mathematical knowledge and a large number of software applications, the accuracy of the transfer function model, and the establishment of a feedback network model; The four models have a certain connection to submit into the relationship, full of metal pollution in the urban topsoil.

Improvement. The analysis of the advantages and disadvantages of available fitting method of least squares method or cubic interpolation, verify the function of the actual size of the error, and further reason to explain the accuracy of the model.

The Promotion of the Model. The four identified models, especially for the analysis of the transfer function model of the feedback network model for the propagation characteristics and evolution of model has a wide range of applications can be applied to the propagation characteristics of the atmospheric pollutants, water pollutants and evolution analysis is a groundbreaking.

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